



Sanjay Ghodawat University, Kolhapur  
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2018-19  
EXM/P/09/01

Year and Program:  
2018-19

School of Technology

Department of FY B.Tech

Course Code: FYT122

Course Title: Complex  
Numbers and Calculus

Semester – II

Day and Date: Monday  
20th May, 2019

ESE

Time: 3 hrs Max Marks: 100  
10-30 to 1-30 PM

Instructions:

- 1) All questions are compulsory.
- 2) Use of non-programmable calculator is allowed.
- 3) Figures to the right indicate full marks.

Q1 Solve the following

Marks Blooms COs  
Levels

a) Prove that,  $(\sqrt{3} + i)^{14} + (\sqrt{3} - i)^{14} = 2^{14}$ .

07 L1 CO1

OR

a) Expand  $\sin 7\theta$ ,  $\cos 7\theta$  and hence find  $\tan 7\theta$ .

07 L1 CO1

b) If  $\tanh \frac{u}{2} = \tan \frac{x}{2}$ , prove that, i)  $\sinh u = \tan x$ ,

08 L2 CO2

ii)  $\cosh u = \sec x$ , iii)  $u = \log \tan \left( \frac{\pi}{4} + \frac{x}{2} \right)$

OR

b) If  $\sin(\theta + i\phi) = (\cos \alpha + i \sin \alpha)$ , prove that,

08 L2 CO2

$\cos^4 \theta = \sin^2 \alpha$

Q2 Solve the following

a) Using Comparison test, discuss the convergence of,

07 L2 CO3

$$\sum_{n=1}^{\infty} \left( \frac{n^3 - 5n^2 + 7}{n^5 + 4n^4 - n} \right)^{\frac{7}{3}}$$

OR

a) Using Cauchy's  $n^{\text{th}}$  root test, discuss the convergence of,

07 L2 CO3

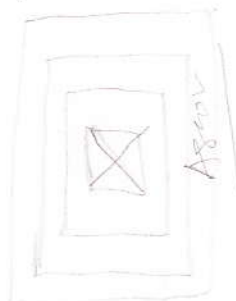
$$\frac{1}{2} + \frac{2}{3}x + \left( \frac{3}{4} \right)^2 x^2 + \left( \frac{4}{5} \right)^3 x^3 + \dots$$

b) Evaluate  $\int_0^1 \frac{x dx}{\sqrt{\log \left( \frac{1}{x} \right)}}$

08 L3 CO4

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- b) Evaluate  $\int_0^2 x^3 \sqrt{2-x} dx$  08 L3 CO
- Q3 Solve any Two of the following**
- a) Solve,  $x^{10} + 11x^5 + 10 \equiv 0$ . 08 L2 CO
- b) Prove that, i)  $\sinh^{-1} z = \log(z + \sqrt{z^2 + 1})$  08 L1 CO  
 ii)  $\tanh^{-1} z = \frac{1}{2} \log \left( \frac{1+z}{1-z} \right)$ .
- c) Using D'Alembert's ratio test, discuss the convergence of  $\frac{x}{1.2} + \frac{x^2}{3.4} + \frac{x^3}{5.6} + \dots, x > 0$ . 08 L2 CO
- d) Evaluate  $\int_0^\infty \frac{x^7(1-x^{12})}{(1+x)^{28}} dx$ , (put  $x = \frac{t}{(1-t)}$ ). 08 L3 CO4
- Q4 Solve any Two of the following**
- a) Solve  $\frac{dy}{dx} = \frac{y}{2y \log y + y - x}$  09 L3 CO5
- b) Solve  $\sin 2x \frac{dy}{dx} = y + \tan x$  09 L3 CO5
- c) In a circuit containing inductance L, resistance R and voltage E, the current i is given by,  $L \frac{di}{dt} + Ri = E$ . If L=640 Henrys, R=250 Ohms, E=500 Volts and i=0 at t=0, find the time that elapses before the current reaches 90% of its maximum value. Show that the current will approach to 2Amps when  $t \rightarrow \infty$ . 09 L3 CO5
- Q5 Solve any Two of the following**
- a) Prove that,  $\log \sec x = \frac{x^2}{2} + \frac{x^4}{12} + \frac{x^6}{45} + \dots$ . 09 L2 CO6
- b) Expand  $x^5 - x^4 + x^3 - x^2 + x - 1$  in powers of  $(x-1)$  and hence find  $f\left(\frac{11}{10}\right)$ . 09 L2 CO6
- c) If  $\lim_{x \rightarrow 0} \frac{x(1+a \cos x) - b \sin x}{x^3} = 1$ , find a and b. 09 L2 CO6

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**Q6 Solve any Three of the following**

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|----|--|----|----|----|
| a) | Solve, $x \frac{dy}{dx} + y = x^3 y^6$ .   | 06 | L3 | CO |
| b) | Show that the system of confocal and coaxial parabolas, $y^2 = 4a(x + a)$ , is self-orthogonal.                                  | 06 | L3 | CO |
| c) | Apply Taylor's theorem to find approximately the value of $f\left(\frac{11}{10}\right)$ , where $f(x) = x^3 + 3x^2 + 15x - 10$ . | 06 | L2 | CO |
| d) | Evaluate $\lim_{x \rightarrow 0} \left( \frac{e^x \sin x - x - x^2}{x^2 + x \log(1-x)} \right)$ .                                | 06 | L2 | CO |

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